Discussion: A Supply and Demand Approach to Equity Pricing

Betermier, Calvet, and Jo

Harjoat S. Bhamra

Imperial College Business School

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- Can we use economics to understand data on equity returns?
    - Firm cash flows do not just fall from the sky
    - The pricing of cashflows reflects investors' objectives if portfolios are chosen optimally – the SDF does not just fall from the sky

$$V_n = \frac{E^{\mathbb{Q}}[CF_n]}{1 + r^f} \tag{1}$$

- Would like to use the above basic valuation framework to understand observed prices, returns and excess returns where CF_n and mapping from $\mathbb{P}$ to $\mathbb{Q}$ are determined endogenously
- CF_n ought to depend on optimal investment decisions
- Mapping from $\mathbb{P}$ to $\mathbb{Q}$ depends on optimal portfolio decisions + market clearing
- Many papers take CF_n or $d\mathbb{Q}/d\mathbb{P}$ (martingale component of SDF) as exogenous

This paper

- Determines CF_n and $d\mathbb{Q}/d\mathbb{P}$ endogenously
- Two-date (static) general equilibrium model
- Tries to connect model's implications to data
  - I shall focus on model's cross-sectional implications

Model

- Two dates, one period, $t \in \{0, 1\}$
- Mean-variance representative agent
- N firms with decreasing returns to scale
- Exogenous shock to wealth of representative agent not a linear combination of shocks to stock returns $\Rightarrow$ incomplete markets.
- No arbitrage $\Rightarrow \exists$ strictly positive SDF Λ (but not unique)

$$\Lambda = \frac{M}{1 + r_f}, \; E[M] = 1 \tag{2}$$

$$M = \frac{d\mathbb{Q}}{d\mathbb{P}} \tag{3}$$

Firms I

- N firms
- Firm n produces cashflow CF_n

$$CF_n = (a_{CF,n} + z_n)\sigma_{CF,n} K_n, \tag{4}$$

where $E[z_n] = 0$ and $Var[z_n] = 0$

- Value of firm n, V_n obtaining via exploiting no arb $\Rightarrow \exists\, \mathbb{Q}$

$$V_n = \frac{E^{\mathbb{Q}}[CF_n]}{1 + r_f} \tag{5}$$

- Return on firm n

$$r_n = \frac{CF_n}{V_n} - 1 \tag{6}$$

  - Physical expected return

$$\mu_n = E[r_n] = \frac{E[CF_n]}{V_n} - 1 \tag{7}$$

$$= a_{CF.n}\sigma_{CF,n}\frac{K_n}{V_n} - 1 \tag{8}$$

Firms II

- Risk-neutral expected return $E^{\mathbb{Q}}[r_n] = r_f \Rightarrow E^{\mathbb{Q}}[z_n] = -(\mu_n - r_f)/\sigma_n \Rightarrow 1 + \mu_n = a_{CF,n}\sigma_n \Rightarrow V_n = \frac{\sigma_{CF,n}}{\sigma_n}K_n$

$$\boxed{1 + \mu_n = a_{CF,n}\sigma_n}$$

$$\boxed{V_n = \frac{\sigma_{CF,n}}{\sigma_n}K_n}$$

  - Only need to determine volatilities and levels of capital to get equilibrium expected returns and prices.

Firms III

Details

$$V_n = \frac{E^{\mathbb{Q}}[CF_n]}{1+r_f} \tag{9}$$

$$= \frac{\left(a_{CF,n} - \frac{\mu_n - r_f}{\sigma_n}\right)\sigma_{CF,n}K_n}{1+r_f} \tag{10}$$

$$\tag{11}$$

$$1+\mu_n = \frac{E[CF_n]}{V_n} - 1 \tag{12}$$

$$1+\mu_n = (1+r_f)\frac{a_{CF,n}}{a_{CF,n} - \frac{\mu_n - r_f}{\sigma_n}} \tag{13}$$

$$1+\mu_n = a_{CF,n}\sigma_n \tag{14}$$

Representative Agent & Portfolio Choice

- Mean-variance objective function

$$E[W_1] - \frac{1}{2W_0}\gamma Var[W_1] \tag{15}$$

- Budget constraint

$$W_1 = W_0[(1 + r_f) + \boldsymbol{\omega}^\top(\boldsymbol{r} - r_f\mathbf{1}) + \underbrace{R_\theta}_{\text{exogenous shock to wealth}}] \tag{16}$$

- Purpose of exogenous shock – generate static hedging demand to distort security market line without technical complications of intertemporal hedging demand – Merton ICAPM effects without the I and without the maths
- Optimal portfolio weights, $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)^\top$

$$\boldsymbol{\omega} = \frac{1}{\gamma}\Sigma^{-1}(\boldsymbol{\mu} - r_f\mathbf{1} - \gamma\sigma\boldsymbol{\theta}_{CF}) \tag{17}$$

Optimal Investment – Implications

Optimal level of capital stock

$$\boxed{K_n^{\eta} = \frac{\sigma_{CF,n}}{\sigma_n}}$$

Only need to determine volatilities to solve for eqm

- In the data, size and book-to-market are distinct factors
- In the model, they are identical

$$\frac{V_n}{K_n} = K_n^{\eta} = \frac{\sigma_{CF,n}}{\sigma_n} \tag{18}$$

No distinction between size and book-to-market

Market clearing I

- Representative agent's risky portfolio weights must equal firm values relative to market value.

$$\frac{\omega_n}{\sum_{n=1}^{N} \omega_n} = \frac{V_n}{\sum_{n=1}^{N} V_n} \tag{19}$$

$$\Rightarrow \tag{20}$$

$$\omega_n = \text{constant} \times V_n \tag{21}$$

$$\frac{1}{\gamma}\Sigma^{-1}(\boldsymbol{\mu} - r_f \mathbf{1} - \gamma\sigma\boldsymbol{\theta}_{CF}) = \text{constant} \times \left(\frac{\sigma_{CF,1}}{\sigma_1}K_1, \ldots, \frac{\sigma_{CF,N}}{\sigma_N}K_N\right)^{\top} \tag{22}$$

$$\frac{1}{\gamma}\Sigma^{-1}(\boldsymbol{\mu} - r_f \mathbf{1} - \gamma\sigma\boldsymbol{\theta}_{CF}) = \text{constant} \times \left(\left(\frac{\sigma_{CF,1}}{\sigma_1}\right)^{1+\frac{1}{\eta}}, \ldots, \left(\frac{\sigma_{CF,N}}{\sigma_N}\right)^{1+\frac{1}{\eta}}\right)^{\top} \tag{23}$$

Market clearing II

$$((a_{CF,1} - \gamma\theta_{CF,1})\sigma_1 - (1 + r_f), \ldots, (a_{CF,N} - \gamma\theta_{CF,N})\sigma_N - (1 + r_f))^\top \quad (24)$$

$$= \text{constant} \times \gamma\Sigma \left(\left(\frac{\sigma_{CF,1}}{\sigma_1} \right)^{1+\frac{1}{\eta}}, \ldots, \left(\frac{\sigma_{CF,N}}{\sigma_N} \right)^{1+\frac{1}{\eta}} \right)^\top \quad (25)$$

- Special case: zero correlations

$$(a_{CF,n} - \gamma\theta_{CF,n})\sigma_n - (1 + r_f) = \text{constant} \times \gamma\, \sigma_{CF,N}^{1+\frac{1}{\eta}} \sigma_n^{\frac{1}{\eta}-1} \quad (26)$$

  - Easy to solve numerically
- In general

$$\sigma_n = f(\boldsymbol{a}_{CF} - \gamma\boldsymbol{\theta}_{CF}, \boldsymbol{\sigma}_{CF}, \boldsymbol{\rho}_{CF}, r_f, \eta, \gamma, N) \quad (27)$$

  - Return volatility as function of exogenous variables

What are the conditions for getting a two factor model? I

- First factor should be unexpected component of market return
- Second factor should be unexpected component of SMB portfolio (equivalent to HML in this model, so no distinct third factor)

Why not exploit the theoretical model by transforming this into a well defined mathematical problem?

What are the conditions for getting a two factor model? II

- Can we write M from $\Lambda = M/(1+r_f)$ in the following form?

$$M = 1 + (R_M - E[R_M]) + (R_{SMB} - E[R_{SMB}]) + \text{zero mean residual}, \quad (28)$$

where

$$R_M = \sum_{n=1}^{N} \omega_n r_n, \quad (29)$$

and

$$R_{SMB} = \sum_{\text{smallest } 10\%} r_n - \sum_{\text{largest } 10\%} r_n \quad (30)$$

Ability to do this will depend on numbers chosen for exogenous variables. But which ones matter the most? Finding the SDF will allow us to answer this question.

Model Implied SDF I

$$\Lambda = \frac{M}{1 + r_f}, \; E[M] = 1. \tag{31}$$

- Assume $M = 1 + \sum_{n=1}^{N} b_n z_n + b_\theta(R_\theta - E[R_\theta]) + \epsilon$, where $E[\epsilon z_n] = E[\epsilon(R_\theta - E[R_\theta])] = 0$ and $E[\epsilon] = 0$.
- Now $E[M z_j] = -\lambda_j$ for $j \in \{1, \dots, N\}$ and so we have N equations to determine $N + 1$ unknowns

$$b_j + \sum_{n \neq j} b_n \rho_{jn} + b_\theta \theta_{CF,j} = -\lambda_j, \; j \in \{1, \dots, N\} \tag{32}$$

Model Implied SDF II

- Matrix form

$$\Omega \boldsymbol{b} = -\boldsymbol{\lambda} - b_\theta \boldsymbol{\theta}_{CF}, \tag{33}$$

where Ω is the following N by N matrix

$$\Omega = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \dots & \rho_{1N} \\ \rho_{21} & 1 & \rho_{23} & \dots & \rho_{2N} \\ \rho_{31} & \rho_{32} & 1 & \dots & \rho_{3N} \\ \vdots & & & \ddots & \\ \vdots & & & & \\ \rho_{N1} & \rho_{N2} & \rho_{N3} & \dots & 1 \end{pmatrix}. \tag{34}$$

Model Implied SDF III

- For arbitrary $b_\theta \in \mathbb{R}$, we have

$$\boldsymbol{b} = -\Omega^{-1}\boldsymbol{\lambda} - b_\theta \Omega^{-1}\boldsymbol{\theta}_{CF}, \tag{35}$$

$$M = 1 - \boldsymbol{\lambda}^\top \Omega^{-1}\boldsymbol{z} + b_\theta(R_\theta - E[R_\theta] - \Omega^{-1}\boldsymbol{\theta}_{CF}) + \epsilon \tag{36}$$

- Special case: zero correlations, i.e. $\Omega = I$

$$M = 1 - \boldsymbol{\lambda}^\top \boldsymbol{z} + b_\theta(R_\theta - E[R_\theta] - \boldsymbol{\theta}_{CF}) + \epsilon \tag{37}$$

  - Require bounds on $\boldsymbol{z}$ and ϵ to ensure $M > 0$
- Need $\theta_{CF,n}$ to switch sign as move from small to big firms.

Cross-sectional asset pricing

- Are the insights any different from Merton ICAPM?
- Need to assume a particular structure for $\boldsymbol{\theta}_{CF}$, but what does the R_θ represent?

Conclusions

- Simple and tractable theoretical framework for studying asset returns.
- Need to explore model more fully to generate new insights beyond Merton's ICAPM.
- Take a stance on what R_θ might be – otherwise hard to falsify model