Risk Premia, Subjective Beliefs, and Forward Guidance

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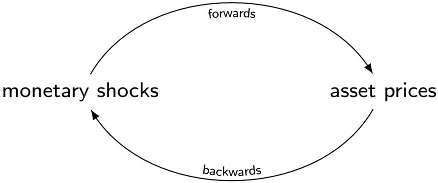
May 29, 2024

Agenda

1. Inverse Problems
2. Paper Summary
3. Comments

Identify Monetary Shocks from Asset Prices



- **Going forwards** monetary shocks impact asset prices
- **Going backwards** infer monetary shocks from asset prices
- Going backwards is an example of an **inverse problem**

A Brief History of Inverse Problems

Definition

An inverse problem in science is the process of calculating from a set of observations the causal factors that produced them

- **Schuster 1882**

  "find out the shape of a bell by means of the sounds which it is capable of sending out"

- **Kac 1966:** "Can one hear the shape of a drum?"

  "Inverse problems are crucial in many scientific disciplines, as they allow us to infer the internal structure of an object from external observations." - Kac, 1966

- **Financial Economics:**
  - Breeden & Litzenberger 1978 – inferring $\mathbb{Q}$ from option prices
  - Ross Recovery – inferring $\mathbb{P}$ and SDF from asset prices
  - Bhamra, Francischello & Martínez-Toledano (2023) – Inferring distribution of risk aversion from dividend strips –

Insights from History of Inverse Problems

- **Gordon & Webb 1996:** You can't hear the shape of a drum
- **Borovicka, Hansen & Scheinkman 2016** Ross Recovery recovers $\hat{\mathbb{P}}$, the long-run risk-neutral distribution, not $\mathbb{P}$
- Most inverse problems are ill-posed
  - **Ill-posed**: inverse problems are almost always ill-posed – solution not unique
- When not ill-posed, they are unstable
  - **Unstable**: even when a unique solution exists for perfect data, it is typically unstable to perturbations in the observed outputs – real-life observations are always partial and corrupted by noise

Paper's Contributions

- Demonstrate that the inverse problem of inferring monetary shocks from asset prices is **ill-posed** in general
- Identify assumptions under which the problem becomes **well-posed** (emphasis on 2 and 3)
    1. short-rate shocks: unexpected changes to the short-term interest rate
    2. forward-guidance shocks: unexpected modifications to the expected future rate path
    3. uncertainty shocks: unexpected changes in uncertainty about future rates
- Required assumption is **long-run neutrality**: risk prices are independent of monetary shocks
- Develop non-parametric test of long-run neutrality.
    - Long-run neutrality does **not** hold empirically
- Interpret long-run neutrality in standard asset pricing models
    - expected consumption growth independent of monetary policy
    - output growth volatility and stock market volatility independent of monetary policy

Economic Structure

- $\mathbb{P}$ – subjective probability measure
- $\mathbb{P}^*$ – risk-neutral measure
- $\hat{\mathbb{P}}$ – long-run risk-neutral measure

State variables

$$d\overbrace{X_t}^{\text{n-dim}} = \mu(X_{t-})\,dt + \underbrace{\sigma(X_{t-})\,dW_t}_{\text{non-monetary } X \text{ shocks}} + \underbrace{\xi_t dM_t}_{\text{monetary-}X\text{ shocks}}, \mathbb{P}$$

Monetary-X shocks arrive according to a Poisson process with rate $\lambda(X_{t-})$

SDF

The SDF process S is given by

$$\frac{dS_t}{S_{t-}} = -r(X_{t-})\,dt - \pi(X_{t-})\cdot dW_t + (\exp[\kappa(X_t, X_{t-})] - 1)dM_t - \chi_S(X_{t-})\,dt, \mathbb{P}$$

Assumptions

$X = (X_t)_{t\geq 0}$ is observable and asset prices are observable. Suppose a monetary shock arrives at time τ. Then $r(X_\tau)$ at time τ and $\mathbb{E}[r(X_\tau) \mid X_{\tau-}]$ are observable.

Monetary-X shocks are jumps and are therefore observable

First Aim of the Paper

Definition 1

Suppose the central bank intervenes at time τ.
The **short rate** shock is given by

$$z_\tau^0 := r(X_\tau) - \mathbb{E}\left[r(X_\tau) \mid X_{\tau-}\right]$$

The shocks to the **expected future short rates** are given by

$$z_\tau^T := \mathbb{E}\left[r(X_{\tau+T}) \mid X_\tau\right] - \mathbb{E}\left[r(X_{\tau+T}) \mid X_{\tau-}\right], \quad T > 0.$$

The shocks to the **distribution of future short rates** are given by

$$p_\tau^T(\mathrm{r}) := \mathbb{P}\left\{r(X_{\tau+T}) \leq \mathrm{r} \mid X_\tau\right\} - \mathbb{P}\left\{r(X_{\tau+T}) \leq \mathrm{r} \mid X_{\tau-}\right\}, \quad T > 0$$

z_τ^0 is observable by assumption.

Question

Under which assumptions can we identify z_τ^T and $p_\tau^T(\mathrm{r})$?

$\hat{\mathbb{P}}$ – long-run risk-neutral measure

Definition

$e(\cdot)$ is an eigenfunction of the SDF process S and $\exp(-\lambda T)$ is the associated eigenvalue if

$$\mathbb{E}\left[\left.\frac{S_{t+T}}{S_t} e\left(X_{t+T}\right)\right| X_t\right] = \exp(-\lambda T) e(X_t)$$

- Many $e(\cdot)$'s and λ's
- An eigenfunction of the SDF is a payoff function which can be valued by a constant discount rate λ
- Assets with eigenfunction payoffs are easy to value!

$$\text{Perron-Frobenius Theorem} \Rightarrow \frac{S_{t+T}}{S_t} = \exp(-\lambda T) \frac{e\left(X_t\right)}{e\left(X_{t+T}\right)} \frac{H_{t+T}}{H_t},$$

- H is a martingale under $\mathbb{P}$
- Choose smallest possible constant discount rate λ (ensuring it's strictly positive is just a tranversality condition)
  Underlying Economics
- Smallest possible λ is yield on infinite-maturity zero-coupon bond

Definition 1

The long-run risk-neutral measure is $\hat{\mathbb{P}}$ defined by the martingale associated with the smallest possible constant discount rate $\hat{\mathbb{P}}(F) := \mathbb{E}\left[1_F H_t\right], \quad \forall F \in \mathcal{F}_t$.

Theoretical Results

Lemma 1

The econometrician observes $\hat{\mathbb{P}}\{r(X_{\tau+T}) \leq \mathrm{r} \mid X_\tau\}$ for every r and every τ, T.

We can recover $\hat{\mathbb{P}}$ from asset prices [Borovicka, Hansen & Scheinkman (2016)].

Definition 2

We say that monetary policy possesses **Long-Run Neutrality** if
(i) The evolution of $d\log(H_t)$ is independent of the monetary shock dM_t;
(ii) The evolution of $d\log(H_t)$ only depends on monetary-insensitive economic states $X_t^\perp$, where monetary-insensitivity is defined by $\mathbb{E}\left[f\left(X_{\tau+T}^\perp\right) \mid X_\tau\right] - \mathbb{E}\left[f\left(X_{\tau+T}^\perp\right) \mid X_{\tau-}\right] = 0$ for all bounded functions $f(\cdot)$, all $T > 0$, and all monetary announcement dates τ.

Proposition 1

Suppose Long-Run Neutrality holds. Suppose further that

$$\mu(x) = A_0 + Ax$$
$$\sigma(x) = B$$
$$r(x) = \rho_0 + \rho \cdot x$$

Then, the forward-guidance shocks $\left(z_\tau^T\right)_{T\geq 0}$ are identified from asset price data alone.

Empirical Results: Non-Parametric Test of Long-Run Neutrality

- Alvarez and Jermann (2005): return of the growth-optimal portfolio in excess of a long-maturity bond identifies the martingale H in the pricing kernel

$$\log\left(R^{*}_{t,t+\Delta}\right) - \log\left(R^{\infty}_{t,t+\Delta}\right) = \log\left(\frac{H_t}{H_{t+\Delta}}\right)$$

$$H \text{ is a martingale under } \mathbb{P} \Rightarrow \mathbb{E}_t\left[\frac{R^{*}_{t,t+\Delta}}{R^{\infty}_{t,t+\Delta}}\right] = 1$$

$$\text{Long-Run Neutrality} \Rightarrow \ln\left(R^{*}_{t,t+\Delta}\right) - \ln\left(R^{\infty}_{t,t+\Delta}\right) \text{ independent of monetary policy shocks}$$

Panel A. Monetary policy decision announcements

	N	Mean	SE(mean)	SD	Skew	Kurtosis
$(-24 \text{ h}, -15 \text{ m})$	210	26.0	8.2	118.2	1.8	18.6
$(-15m, +15m)$	210	-1.3	4.1	59.5	-1.5	11.3
$(-15m, +24h)$	210	-20.1	12.3	178.2	-0.8	6.0
Panel B. Press conferences						
	N	Mean	SE(mean)	SD	Skew	Kurtosis
$(-24 \text{ h}, -15 \text{ m})$	70	19.8	9.1	76.1	0.0	3.7
$(-15m, +15m)$	70	9.4	4.7	39.1	1.2	5.5
$(-15m, +60m)$	70	2.4	8.2	68.4	0.2	4.3
$(+60 \text{ m}, +24 \text{ h})$	70	-43.5	19.2	160.5	-1.0	5.7

Table 1. Summary statistics. The table reports summary statistics for log returns on S&P500 E-mini futures minus log returns on 30 -year Treasury bond futures in various windows around scheduled monetary FOMC decision announcements and press conferences. A futures "return" in window $(t, t+\Delta)$ is defined as $F_{t+\Delta}/F_t$, where F denotes the futures price. The sample covers FOMC meetings from 1997:09 through 2023:12, with press conferences introduced in 2011.

- Senstive to monetary policy news

H and Asset Pricing Models

- In Bansal & Yaron model with EIS one and no stochastic vol
  $\log C_{t+1} = \log C_t + \alpha \cdot (X_{t+1} - X_t)$
  $X_{t+1} = A_0 + AX_t + B\Delta W_{t+1}, \quad \Delta W_{t+1} \sim \mathrm{Normal}(0, I)$
  Long-run neutrality $\Rightarrow$ no monetary shocks in ΔW_{t+1}

What is the Paper's Main Result?

- What do we learn from knowing that long-run neutrality is violated?
- Do we need to start building asset pricing models where
  - aggregate output is related to monetary policy?
  - risk prices are affected by monetary policy?
  - or both?
- Is aggregate consumption really that important for understanding asset price dynamics ?
- Perhaps other economic variables are more important. Do the way in which long-run neutrality is violated tell us what they are?

Tell us more about H!

- Dynamics of H are key
  - Why is ok to assume H is driven solely by market returns. What about Fama-French factors?
  - Is $\frac{R^*_{t,t+\Delta}}{R^\infty_{t,t+\Delta}}$ a martingale in the data?
  - Models
    - Directly compare your H with models that feature an FOMC announcement premium, e.g. Ai, Bansal, & Yaron
    - Can you reject Ai, Bansal, & Yaron?
  - Empirics
    - How is H related to monetary policy uncertainty?

H is not the usual martingale we see in the SDF

- Usual decomposition

$$\frac{S_{t+T}}{S_t} = e^{-\int_t^{t+T} r(X_{u-})du} \frac{M_{t+T}}{M_t},$$

  - M is a martingale under $\mathbb{P}$ and changes measure from $\mathbb{P}$ to $\mathbb{P}^*$

- Decomposition used in this paper

$$\frac{S_{t+T}}{S_t} = \exp(-\lambda T) \frac{e(X_t)}{e(X_{t+T})} \frac{H_{t+T}}{H_t},$$

  - $\lambda > 0$ is yield in infinite maturity bond
  - H is a martingale under $\mathbb{P}$ and changes measure from $\mathbb{P}$ to $\hat{\mathbb{P}}$

$$e^{-\int_t^{t+T} r(X_{u-})du} \frac{M_{t+T}}{M_t} = \exp(-\lambda T) \frac{e(X_t)}{e(X_{t+T})} \frac{H_{t+T}}{H_t}$$

- What does long-run neutrality imply for $d \ln M_t$?

$$\frac{dM_t}{M_t} = \frac{dH_t}{H_t} + de(X_t) - \mathbb{E}_t[de(X_t)]$$

  - Can the above expression tell us anything useful?

Monetary vs Real?

- What's monetary about this economy?
- No economic distinction between monetary and non-monetary shocks

State variables

$$d\overbrace{X_t}^{\text{n-dim}} = \mu\left(X_{t-}\right)dt + \underbrace{\sigma\left(X_{t-}\right)dW_t}_{\text{non-monetary } X \text{ shocks}} + \underbrace{\xi_t dM_t}_{\text{monetary-}X\text{ shocks}}, \mathbb{P}$$

- Just a mathematical distinction
- No inflation

More Structure? I

- Would more structure help with the inverse problem?
- Look at Fed Funds Rate

$$di_t = \mu_{t-}dt + \sigma_{t-}dZ_t + \sigma_{N,t-}dN_t$$

- σ_{t-} is small
- dN_t jumps up by 1 every FOMC meeting date.
- Why not smooth out the jumps and assume

$$di_t = \mu_{t-}dt + \sigma_{t-}dZ_t$$

- current value of μ is unknown
- Fed generally adjusts in 25 b.p. increments with changes less than or equal to 100 b.p. FOMC meets 8 times per year, so 25 bp every meeting means 2% increase per year.

$$\mu_t \in \{-0.08, -0.06, -0.04, -0.02, 0, 0.02, 0.04, 0.06, 0.08\}$$

More Structure? II

- From investors' viewpoint

$$di_t = \tilde{\mu}_{t-} dt + \sigma_{t-} d\tilde{Z}_t,$$
$$\tilde{\mu}_{t-} = \sum_{i=1}^{9} \pi_{i,t-} a_i$$

$$a_i \in \{-0.08, -0.06, -0.04, -0.02, 0, 0.02, 0.04, 0.06, 0.08\}$$

- Obtain $\pi_{i,t-}$'s from Wonham filter. Then get $\tilde{\mathbb{E}}_t[i_u]$.
- Key (and difficult) question: signal structure – should this be based in macroeconomic signals and prices?

Summary of Comments

- Very interesting application of operator methods
  - Results are non-trivial – emphasize this and connect to wider literature on inverse problems
- Paper needs a clear punchline
- How is long-run neutrality is violated and what does this tell us about the monetary policy factors driving H?
- Do violations of long-run neutrality help us reject any models?
- The model is not very monetary
- Can more structure help with this particular inverse problem?

Appendix I

$$\mathbb{E}_t\left[\frac{S_{t+dt}}{S_t}e(X_{t+dt})\right] = e^{-\lambda dt}e(X_t) \tag{1}$$

$$\mathbb{E}_t\left[\left(1+\frac{dS_t}{S_t}\right)(e(X_t)+de(X_t))\right] = (1-\lambda dt)e(X_t)+o(dt) \tag{2}$$

$$e(X_t)+\mathbb{E}_t[de(X_t)]+\mathbb{E}_t\left[\frac{dS_t}{S_t}e(X_t)\right] = (1-\lambda dt)e(X_t)+o(dt) \tag{3}$$

$$e(X_t)+\mathbb{E}_t[de(X_t)]+\mathbb{E}_t\left[\frac{dS_t}{S_t}e(X_t)\right]+\mathbb{E}_t\left[\frac{dS_t}{S_t}de(X_t)\right] = (1-\lambda dt)e(X_t)+o(dt) \tag{4}$$

$$\mathbb{E}_t[de(X_t)]+\mathbb{E}_t\left[\frac{dS_t}{S_t}e(X_t)\right]+\mathbb{E}_t\left[\frac{dS_t}{S_t}de(X_t)\right] = -\lambda e(X_t)dt+o(dt) \tag{5}$$

$$\mathbb{E}_t[de(X_t)]-r(X_t)e(X_t)dt+\mathbb{E}_t\left[\frac{dS_t}{S_t}de(X_t)\right] = -\lambda e(X_t)dt+o(dt) \tag{6}$$

$$\mathbb{E}_t\left[\frac{de(X_t)}{dt}\right]+\frac{1}{dt}\mathbb{E}_t\left[\frac{dS_t}{S_t}de(X_t)\right]-(r(X_t)-\lambda)e(X_t)=0 \tag{7}$$

$$\mathbb{E}_t^*\left[\frac{de(X_t)}{dt}\right]-(r(X_t)-\lambda)e(X_t)=0$$

Appendix II

$$e^{-(r(X_t)-\lambda)dt}\left(1+\frac{dM_t}{M_t}\right)\left(1+\frac{de_t}{e_t}\right)=\left(1+\frac{dH_t}{H_t}\right) \tag{8}$$

$$\left(1-(r(X_t)-\lambda)dt+o(dt)\right)\left(1+\frac{dM_t}{M_t}\right)\left(1+\frac{de_t}{e_t}\right)=\left(1+\frac{dH_t}{H_t}\right) \tag{9}$$

$$\left(1-(r(X_t)-\lambda)dt+o(dt)\right)\left(1+\frac{dM_t}{M_t}+\frac{de_t}{e_t}+\frac{dM_t}{M_t}\frac{de_t}{e_t}\right)=\left(1+\frac{dH_t}{H_t}\right) \tag{10}$$

$$\left(1+\frac{dM_t}{M_t}+\frac{de_t}{e_t}+\frac{dM_t}{M_t}\frac{de_t}{e_t}\right)-(r(X_t)-\lambda)dt=\left(1+\frac{dH_t}{H_t}\right)+o(dt) \tag{11}$$

$$\frac{dM_t}{M_t}+\frac{de_t}{e_t}+\frac{dM_t}{M_t}\frac{de_t}{e_t}-(r(X_t)-\lambda)dt=\frac{dH_t}{H_t}+o(dt) \tag{12}$$

$$\frac{dM_t}{M_t}-\mathbb{E}_t\left[\frac{dM_t}{M_t}\right]=\frac{dH_t}{H_t}-\mathbb{E}_t\left[\frac{dH_t}{H_t}\right]-\mathbb{E}_t[de_t]+de_t \tag{13}$$

$$\frac{dM_t}{M_t}=\frac{dH_t}{H_t}-\mathbb{E}_t[de_t]+de_t \tag{14}$$

$$\frac{dM_t}{M_t}=\frac{dH_t}{H_t}+de_t-\mathbb{E}_t[de_t] \tag{15}$$