Asset Pricing with Optimal Under-Diversification by Elenev and Landvoigt

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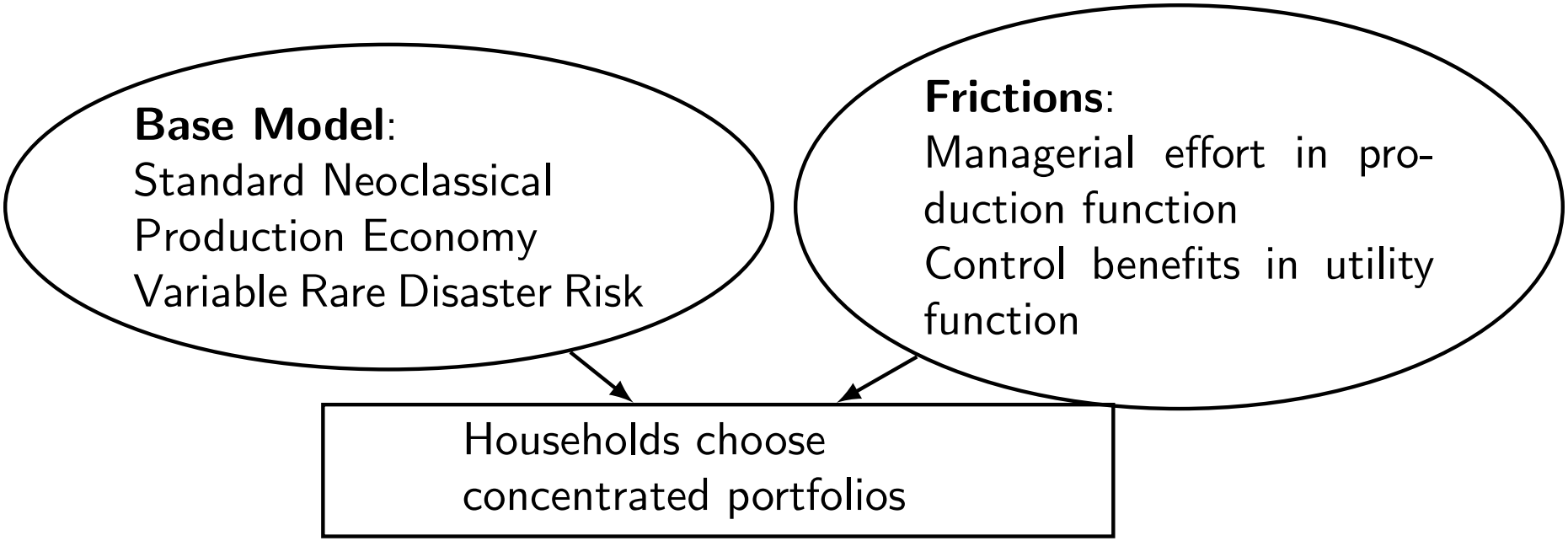
Relatively Sophisticated Asset Pricing Model v. Data

- Model
  - N risky assets, 1 risk-free asset
  - H EZW households with heterogeneous preferences, but identical beliefs
  - Assume dynamically complete markets $\Rightarrow$ perfect diversification across risky assets
  - All agents want to hold the same subportfolio of N risky assets – the market portfolio
  - Just replace the N risky assets with one risky asset
- Data
  - Some households do not own equity
  - Participating households hold different subportfolios of risky assets. The subportfolios are poorly diversified. [Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Campbell (2006), Guiso and Sodini (2013)]
  - Direct holdings often consist of two or three stocks! [Polkovnichenko (2005)]
  - Very wealthy households hold more diversified portfolios of risky assets, but have large holdings in private businesses, Smith, Zidar, and Zwick (2022).
- Even a relatively sophisticated model does not capture household under-diversification

This paper

- Very wealthy households hold more diversified portfolios of risky assets, but have large holdings in private businesses
- Top 10% wealthiest U.S. households own close to 90% of all (public and private) business equity, and almost 50% of these assets are privately held.
- Does fact this help us quantitatively understand the size of the aggregate equity premium and aggregate equity return vol?
- Partially, yes: Exposure to undiversified firm risk can explain approximately 2/5 of the level and 1/5 of the volatility of the equity premium.

Model Summary: Base Model + Frictions



Households

- All households have identical EZW preferences, but not over intermediate consumption, but over over intermediate consumption plus a function of risky asset holdings and "private business" holdings

$$u\left(C_{t,i}, \eta_{t,i}, S_{t+1,i}\right) = C_{t,i} + \psi S_{t+1,i}\sqrt{\eta_{t,i}}$$

- Household i invests in a risky fund split $(1-\eta_{t,i}, \eta_{t,i})$ between a perfectly diversified portfolio of risky assets and a single asset.
- The single asset ("private business" holding) changes randomly every period: this ensures symmetry across households and ensures the wealth distribution is irrelevant for asset prices.
- Only the very wealthy agents are modelled. This is important.

Firms

$$F\left(K_t, L_t, O_t\right) = A_t K_t^{1-\alpha_L} \left(Z_t L_t\right)^{\alpha_L - \alpha_o} \left(Z_t O_t\right)^{\alpha_o}$$

- A_t is total factor productivity (TFP) – level shrinks when disaster hits
- Z_t is labor-augmenting productivity – growth rate shrinks when disaster hits
- K_t capital
- L_t labor
- O_t managerial effort invested in firm by the household that owns the firm outside its undiversified portfolio

How I will think about the economics I

The model is complex and solved numerically. I need an anchor to understand it.

- Asset prices have two components
  - prices across time
  - prices across states
- Decomposing the SDF helps us understand these two components

$$M_{t,t+1} = e^{-r_{t,t+1}} \frac{\mathfrak{M}_{t+1}}{\mathfrak{M}_t} \tag{1}$$

  - $B_{t,t+1}$ is the time-t price of a bond that pays off a unit of consumption at time $t+1$ – price across time
    - $B_{t,t+1}$ is impacted by three aspects of household behavior
    - impatience
    - desire for intertemporal smoothing
    - risk aversion
    - prudence and desire for precautionary savings – least well understood
  - $\mathfrak{M}_t$ is an exponential martingale under $\mathbb{P}$ – relative price of consumption across states for a fixed time t

How I will think about the economics II

  - aversion to static risk
  - aversion to intertemporal risk

- How is this related to the under-diversification of the rich and this paper?
  - $B_{t,t+1}$ and hence demand for precautionary savings impacted by those who hold 'risk-free' assets – everyone! Wealth and consumption distributions will matter.
  - $\mathfrak{M}_t$ impacted by those who hold risky assets – mainly held by rich. Can just focus on them. Wealth and consumption distributions will matter less.

Is precautionary savings important in this model?

- If precautionary savings demand is important for the results in this model, then ignoring the less wealthy agents is not innocuous.
- For modelling assumption to make sense, you would need to show that precautionary demand is determined by the wealthy.

Huge Precautionary Demand

- Risk-free rate

$$r = -\ln \beta + \frac{1}{\text{EIS}} g - \text{PS terms} \tag{2}$$

  - Your calibration: $\beta = 0.76$ (annualized).
  - In an economy with no risk and no growth in consumption $\Rightarrow r = -\ln \beta = 26\%$ p.a.
  - 26% is massive. Your interest rate with growth and risk comes down to $r = 1.11\%$.
  - With full diversification, $r = 7.12\%$
  - Precautionary savings motive in your model is huge
    - disaster risk
    - idiosyncratic risk

- Precautionary savings demand seems fundamental for your results:

  'Our second set of results quantifies the impact of idiosyncratic exposure on aggregate outcomes. Going to a counterfactual world of full diversification by turning off both sources of under-diversification, would lead to **substantially higher risk free interest rates and lower excess returns**. The risk free rate is 6% points higher and the market risk premium is 2.5% points lower in the full diversification case. The volatility of excess returns drops from 17% to 14%. Thus, we find that lack of diversification can explain approximately 40% of the market excess return and close to 20% of its volatility. On the real side, reduced idiosyncratic risk leads to a **smaller precautionary savings motive**, resulting in an economy with a 4% smaller capital stock.,

- You need to

  - Show that precautionary savings demand is disproportionately driven by the wealthy
  - Show that precautionary savings demand of the wealthy is large

Precautionary Savings and the Rich vs the Poor I

- Two agent model, CRRA (different RRA), endowment economy – Bhamra & Uppal (2009)

$$r_t = -\ln\beta + \mathbf{R}_t\mu_Y - \frac{1}{2}\mathbf{R}_t\mathbf{P}_t\sigma_Y^2 \tag{3}$$

- aggregate risk aversion

$$\mathbf{R}_t = \left(\nu_{1,t}\frac{1}{\gamma_1} + \nu_{2,t}\frac{1}{\gamma_2}\right)^{-1} \tag{4}$$

- aggregate prudence

$$\mathbf{P}_t = (1+\gamma_1)\left(\frac{\mathbf{R}_t}{\gamma_1}\right)^2\nu_{1,t} + (1+\gamma_2)\left(\frac{\mathbf{R}_t}{\gamma_2}\right)^2\nu_{2,t} \tag{5}$$

- $\nu_{k,t}$ is consumption-share (not wealth) of agents with RRA γ_k

Precautionary Savings and the Rich vs the Poor II

- Which fraction of aggregate consumption (non-durables and services) is consumed by the wealthiest 10%?

- Guess: it is going to be less than 90%

Other work on the lack of diversification

- This paper focuses on under-diversification of the wealthy
- Sensible if you want to understand risk pricing
- There is much work on lack of diversification in finance and macro
- Don't just cite the macro work

Household Under-Diversification

- More precise information about particular areas based on network-based information acquisition, Cohen, Frazzini, and Malloy (2008)
- Private Benefits of Control, Gorton & Rosen (1995)
- Psychological biases: loyalty [Cohen, (2009)], familiarity [Huberman (2001), Bhamra & Uppal (2019,2021)] , geography [Grinblatt and Keloharju (2001), Bhamra, Uppal, Walden (2023)]
- Portfolio diversification of firms' controlling owners and capital investment: Lyandres, Marchica, Michaely, Mura (2019)

What do we already know?

- Under diversification has the following effects [taken from Bhamra & Uppal (2019) & Bhamra & Uppal (2021)]

- Return on wealth for individual households becomes riskier
- Consumption growth for individual households becomes riskier
- In equilibrium: aggregate consumption growth becomes riskier (effects do not cancel out)
- Price of risk increases, equity risk premium increases
- Individual agents reduces investment in risky assets
- Aggregate growth falls – amplifies welfare losses from lack of diversification by a factor of 4

- What's new in this paper?

Your contribution

- Very serious **quantitative** exercise
- Match many moments
- You quantify how important lack of diversification by the rich is for asset prices.
- The model is complex using high-end solution techniques
- Economic mechanisms need to be clearer

Decompose the SDF I

- Your SDF

$$M_{t,t+i}^{i} = \beta \left(\frac{v^{H}\left(\mathcal{Z}_{t+1}\right)}{v^{H}\left(\mathcal{Z}_{t}\right)} \right)^{1-\gamma} \left(\frac{v^{H}\left(\mathcal{Z}_{t+1}\right)}{CE_{t}} \right)^{\gamma-\sigma} \left(r_{t+1,i}^{H}\right)^{-\sigma}$$

$$r_{t+1,i}^{H} = \frac{W_{t+1,i}}{W_{t,i}}$$

$$\mathcal{M}_{t,t+1}^{i} = M_{t,t+i}^{i} \frac{n_{t+1,i}^{\eta}}{n_{t+1,i}}$$

- Decompose SDF into

$$M_{t,t+i}^{i} = B_{t,t+1} \frac{\mathfrak{M}_{t+1}^{i}}{\mathfrak{M}_{t}^{i}} \tag{6}$$

Decompose the SDF II

- Martingale $\mathfrak{M}_t^i$ tells us relative price of a unit of consumption across states at the same time.
- Decompose $\mathfrak{M}_t^i$ into underlying factors, so we can understand which risks are priced.

What drives under-diversification?

- A household does not choose which firm to hold outside its diversified portfolio: a firm is randomly assigned every period
- This is for tractability, but other approaches are possible: firm-specific ambiguity aversion – see Bhamra & Uppal (2019)
- You put the number of risky portfolio shares in the utility function via

$$u\left(C_{t,i}, \eta_{t,i}, S_{t+1,i}\right) = C_{t,i} + \psi S_{t+1,i}\sqrt{\eta_{t,i}}$$

- What does this have to do with diversification? Is this a scaling issue?
- Why not use the following?

$$u\left(C_{t,i}, \eta_{t,i}, S_{t+1,i}\right) = C_{t,i} + \psi\sqrt{\eta_{t,i}}$$

Under-diversification & the Wealthy

- Why are your assumptions about the drivers of under-diversification better than:
- Expertise in particular area + attractive returns relative to risk from innovation in area of expertise
- Expertise in particular area $\Rightarrow$ more precise expected return estimates for firms in that area
- Under diversification + skill or luck is what makes you very wealthy!

Conclusion

- Economic mechanisms need to be clearer
  - Decompose your SDF
- Size of precautionary savings demand seems fundamental to quantitative results
- Show that precautionary savings demand is disproportionately driven by the wealthy
  - Which fraction of aggregate consumption (non-durables and services) is consumed by the wealthiest 10%?
- Show that precautionary savings demand of the wealthy is large
- Why are your assumptions about the drivers of under-diversification better than others?