The Optimal Schedules of Incentives and Cash Flows by Feng, Luo, and Westerfield

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The Short-Term vs. Long-Term Dilemma

A Fundamental Question in Corporate Finance

The CEO's Daily Choice

Every day, executives face a choice:

- **Option A**: Actions that boost *today's* earnings
- **Option B**: Investments that build *tomorrow's* value

Which type will compensation contracts encourage?

Short-Term Actions: The Quick Wins

Short-Term Actions (a_t) - Instant Impact

Revenue Boosts:

- Channel stuffing at quarter-end
- Pull forward next quarter's sales
- Aggressive revenue recognition

Cost Cuts:

- Slash marketing budget
- Hiring freeze
- Reduce R&D spending

Result: Earnings jump THIS quarter

Long-Term Investments: Building the Future

Long-Term Investments (q_t) - Future Payoff

Innovation:

- R&D for next-generation products
- Patent portfolio development
- Technology infrastructure

Strategic Assets:

- Brand building campaigns
- Employee training programs
- Customer relationship systems

Result: Earnings improve over YEARS

Example: Pharmaceutical Company CEO's Choice

Option A: Quick Hits

- Raise drug prices 15%
- Cut sales force travel
- Delay factory maintenance

Impact: +\$50M this quarter

Option B: Build Pipeline

- Fund Phase II trials
- Hire top researchers
- FDA relationship building

Impact: +\$500M in 5-10 years

Same present value, different timing!

The Measurement Problem: What We See

This Quarter's Earnings Report

"Earnings: \$2.50 per share (beat by \$0.10)"

But What Caused This?

- CEO pushed hard this quarter? (high a_t)
- Past R&D paying off? (high Q_t)
- Competitors had problems? (lucky noise)
- Some mix of all three?

We cannot decompose the total!

The Fundamental Equation

What Generates Cash Flows

$$dY_t = [\underbrace{a_t}_{\text{instant}} + \underbrace{(r+\delta)Q_t}_{\text{from past}}]dt + \underbrace{\sigma dZ_t}_{\text{noise}}$$

The Key Point

- a_t: This period's short-term effort
- Q_t: Stock of quality from past investments
- Both create **identical** cash flows
- But we observe only the **sum** Y_t

What Exactly is "Quality" Q_t?

Intangible Assets We Can't Measure

- Employee skills
- Customer loyalty
- Brand value
- Innovation pipeline
- Corporate culture
- Supplier relationships
- Process improvements
- Data capabilities

Why We Can't Measure It

- No accounting standards for intangibles
- Highly firm-specific
- Strategic secrecy required

Who Knows What?

The Information Asymmetry

	Cash Flow Y_t	Current Effort a_t	Current Investment q_t	Quality Stock Q_t
Board	✓	×	×	×
CEO	✓	✓	✓	✓

The Dynamic Problem

- Manager builds hidden Q_t over time
- Information asymmetry **accumulates**
- Future performance becomes increasingly opaque

Key Model Assumptions (1/3)

Assumption 1: Equal Productivity

- \$1 of effort $a_t \to$ \$1 of cash flow today
- \$1 of investment $q_t \to$ \$1 of cash flow (PV)
- **No technological bias**

Assumption 2: Perfect Pooling

- Output combines both: $dY_t = (a_t + (r + \delta)Q_t)dt + \sigma dZ_t$
- Cannot separate the components
- Cannot contract on a_t or q_t directly

Key Model Assumptions (2/3)

Assumption 3: CARA Preferences

- Manager has exponential utility
- Can secretly save/borrow at rate r
- Makes continuation value a martingale
- Enables analytical solution

Assumption 4: Limited Contracts

- Pay-performance sensitivity $\alpha_t \in [0, 1]$
- Cannot sell firm to manager
- Cannot make manager bear losses

Key Model Assumptions (3/3)

What These Assumptions Achieve

- Isolate the **pure timing effect**
- Remove any technological advantages
- Focus on agency problem alone
- Enable clean theoretical results

Question: Given only timing differences, which effort type will contracts favor?

The Conventional Wisdom

Why Short-Term "Should" Win

1. **Immediate Feedback**
   - See results right away
   - Clear link: effort $\rightarrow$ outcome
2. **No Information Accumulation**
   - a_t leaves no hidden trace
   - q_t builds hidden Q_t forever
3. **Risk Compensation Today**
   - Agent bears risk now
   - Must compensate immediately

The Surprising Result

But the conventional wisdom is WRONG!

The Paper's Key Finding

Long-term effort is actually **CHEAPER** to incentivize!

The agency problem *helps* fight short-termism!

Preview: Why Long-Term is Cheaper

The Key Insight

1. Risk compensation can be **deferred**
   - Pay for risk in the future
   - Discounting reduces cost
2. The **envelope theorem** applies
   - At optimal short-term incentives
   - Marginal cost of risk is zero
   - But marginal benefit of long-term positive

Let's see how this works mathematically...

Economic Innovation

Key Departure from Literature

1. **Pooling of outputs**: $dY_t = (a_t + (r+\delta)Q_t)dt + \sigma dZ_t$
   - Unlike Gryglewicz et al. (2020): separate signals
   - Unlike Marinovic-Varas (2019): bad long-term action
2. **Both actions are first-best efficient**
   - No technological bias toward either
   - Pure agency-driven duration choice

Central Result

Despite persistent private information, durable effort is *cheaper* to incentivize when incentives are moderate!

Technical Innovation

Methodological Contributions

1. **State variable reduction**: From (W_t, Q_t, S_t) to single z_t

$$z_t = \hat{\mathbb{E}}_t \left[\int_t^\infty e^{-(r+\delta)(s-t)} (r+\delta) \frac{W_s}{W_t} \alpha_s ds \right]$$

2. **Endogenous correlation**: $\eta_t \leq 0$ emerges optimally
3. **Duration analytics**: Link incentive schedules to cash flow autocorrelation

"Adding-up constraint" $\Rightarrow$ *economic trade-offs*

Model Setup

Technology

$$dQ_t = (q_t - \delta Q_t)dt$$
$$dY_t = (a_t + (r+\delta)Q_t)dt + \sigma dZ_t$$

Preferences

$$u(c,a,q) = -\frac{1}{\gamma}e^{-\gamma(c-\frac{\lambda}{2}(a^2+q^2))}$$

Information

- Principal observes: Y_t
- Doesn't observe: a_t, q_t, Q_t
- Hidden savings: S_t

Contract Space

- PPS: $\alpha_t \in [0,1]$
- Compensation:
  $dM_t = (\cdots)dt + \beta_t \sigma dZ_t$

Agent's Optimization

Agent's Problem

$$\max \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} u(c_s, a_s, q_s) ds \right]$$

subject to quality evolution, savings dynamics, and cash flow process

Key: Change of Measure

Agent's true effort $\neq$ recommended $\Rightarrow$ different probability measures

$$d\xi_t = \frac{\Delta a_t + (r+\delta)\Delta Q_t}{\sigma} \xi_t d\hat{Z}_t$$

where ξ_t is the Radon-Nikodym derivative

Deriving the State Variable z_t

First-Order Conditions (evaluated at truth-telling)

$$
\begin{aligned}
[\Delta c]: &\quad -\gamma u = p^S \\
[\Delta a]: &\quad \lambda\gamma a u = -\frac{j^\xi}{\sigma} \\
[\Delta q]: &\quad \lambda\gamma q u = -p^Q
\end{aligned}
$$

Key Insight

Define $z_t := \frac{p_t^Q}{-\gamma r W_t}$ where p_t^Q is the shadow value of quality

Critical: W_t is a Martingale

Hidden savings + CARA $\Rightarrow \mathbb{E}_t[dW_t] = 0$

The Magic of z_t: Integrating Forward

From Shadow Prices to Incentives

Integrating the adjoint equation for p_t^Q:

$$p_t^Q = \hat{\mathbb{E}_t}\left[\int_t^{\infty} e^{-(r+\delta)(s-t)}(-(r+\delta)\gamma r W_s \alpha_s)ds\right]$$

The State Variable

$$z_t = \frac{p_t^Q}{-\gamma r W_t} = \hat{\mathbb{E}_t}\left[\int_t^{\infty} e^{-(r+\delta)(s-t)}(r+\delta)\frac{W_s}{W_t}\alpha_s ds\right]$$

Interpretation

z_t = utility-weighted average of future pay-performance sensitivities!

Agent's Incentive Compatibility

Key State Variables

- W_t: Agent's continuation utility (martingale)
- z_t: Marginal value of quality vs. savings

Incentive Constraints

$$a_t^* = \frac{\alpha_t}{\lambda} \quad \text{(static Euler equation)} \tag{1}$$

$$q_t^* = \frac{z_t}{\lambda} \quad \text{(dynamic Euler equation)} \tag{2}$$

Key insight: Long-term incentives are utility-weighted future short-term incentives!

Principal's Problem: The HJB Equation

Value Function Decomposition

$$V(Q_t, z_t) = Q_t + v(z_t)$$

HJB Equation

$$\begin{aligned} rv(z) = \max_{\alpha,\eta} \Big\{ & \frac{1}{\lambda}(\alpha + z) - \frac{1}{2\lambda}(\alpha^2 + z^2) - \frac{\sigma^2 \gamma r}{2}\alpha^2 \\ & + v'(z)(r+\delta)(z - \alpha + \sigma^2 \gamma r \alpha \eta) \\ & + \frac{1}{2} v''(z)(r+\delta)^2 \eta^2 \sigma^2 \Big\} \end{aligned}$$

First-Order Conditions and Key Result

FOC for α

$$\frac{1}{\lambda}(1-\alpha) - \sigma^2\gamma r\alpha + v'(z)(r+\delta)(\sigma^2\gamma r\eta - 1) = 0$$

FOC for η

$$v'(z)\sigma^2\gamma r\alpha + v''(z)(r+\delta)\sigma^2\eta = 0$$

Key Result

When $v''(z) < 0$ (concavity), we get $\eta < 0$ when $v'(z) < 0$

This negative correlation between output and incentives is crucial!

Boundary Conditions and Reflecting Barriers

At $z = 0$

$$v(0) = 0$$

No future incentives

At $z = 1$

$$v(1) = \frac{1}{\lambda r} - \frac{\sigma^2 \gamma}{2}$$

Maximum future incentives

At the Optimum z^*

- $v'(z^*) = 0$ (maximum)
- $\eta(z^*) = 0$ (no correlation)
- $z^* > \alpha^* = \frac{1}{1+\gamma r \sigma^2 \lambda}$

Proof Sketch: Why $z^* > \alpha^*$?

The Envelope Theorem Argument

At α^*, the marginal value from short-term effort is:

$$\frac{\partial}{\partial \alpha}\left[\frac{1}{\lambda}\alpha - \frac{1}{2\lambda}\alpha^2 - \frac{\sigma^2\gamma r}{2}\alpha^2\right]\Bigg|_{\alpha=\alpha^*} = 0$$

Cost-Benefit of Increasing z from α^*

- **Benefit**: Higher future cash flows $\frac{1}{\lambda}(z - \alpha^*)dt$
- **Cost**: Future risk compensation (discounted!)
- At margin, benefit $>$ cost due to discounting

Result

Optimal contract starts with $z^* > \alpha^*$ (back-loaded)

Key Results: Optimal Policies

Three Regimes

1. **Low z (after success)**: Back-loaded, low power
   - $z^* > \alpha^*$, $\eta = 0$ at optimum
2. **Medium z**: Transition region
   - $\eta < 0$: negative correlation
3. **High z (after failure)**: Front-loaded, high power
   - $\alpha = 1$ binding

Solution Structure: Three Regions

Region 1: $z \in [z^*, z^{**}]$

- Interior solution for α
- $\eta < 0$ (except at z^*)
- Back-loaded when $z < \alpha$

Region 2: $z \in [z^{**}, 1]$

- $\alpha = 1$ (binding constraint)
- $\eta < 0$ still optimal
- Strongly front-loaded

Dynamics

$\mathbb{E}[dz_t] = (r + \delta)(z_t - \alpha_t)dt$, Mean reversion toward α_t

Economic Mechanism

Why are durable incentives cheaper?

1. **Timing of compensation**:
   - Instantaneous: risk compensation paid today
   - Durable: risk compensation paid in future (discounted)
2. **Envelope theorem at α^***
3. **First-order gain from $z > \alpha^*$ at zero marginal cost!**

Implication

Optimal contracts start back-loaded despite persistent private information

Verification: Preventing Double Deviations

Upper Bound on Payoff from Deviation

$$J(W, \Delta S, \Delta Q, z) = W \exp(-\gamma r \Delta S - \gamma r z \Delta Q)$$

Sufficient Condition

Show that:

$$u(\hat{c} + \Delta c, \hat{a} + \Delta a, \hat{q} + \Delta q) - rJ + \mathbb{E}[dJ]/dt \leq 0$$

Key Requirements

- $\eta \leq 0$ (negative correlation)
- $\lambda \sigma^2 \gamma r \geq 1$ (parameter restriction)

Dynamics and Mean Reversion

Cash Flow Dynamics

- $\eta < 0$: Incentives increase after losses
- Creates mean reversion
- Endogenous autocorrelation

Duration Results

$$D_t(\alpha) = \frac{1}{r} \cdot \mathbb{E}_t\left[\frac{F_s(\alpha)}{F_t(\alpha)}\right]$$

Duration decreases with z!

Stochastic Dynamics and Ergodic Distribution

Evolution of z_t

$$dz_t = (r+\delta)(z_t - \alpha_t)dt + (r+\delta)\sigma\eta_t d\hat{Z}_t$$

Long-Run Behavior

- Process stays in $[z^*, 1)$ if starting at z^*
- Negative shocks $\rightarrow$ increase z (tougher incentives)
- Positive shocks $\rightarrow$ decrease z (relaxed incentives)
- Ergodic distribution concentrated away from boundaries

Connection to Cash Flows

Autocorr$(Y_t, Y_{t+h}) < 0$ for intermediate h

Critique 1: Implementation Concerns

The $\alpha_t \in [0,1]$ Restriction

While microfounded, doesn't this severely constrains the contract space?

Issues

- Real contracts use options, restricted stock, clawbacks
- Could implement $\alpha > 1$ with leverage constraints
- Message games could expand feasible set (Zechner, 1996)

Recommendation

Extend to convex compensation schemes:

$$M_t = \phi_0(Y_t) + \phi_1(Y_t)\mathbf{1}_{\{Y_t > K_t\}}$$

Critique 1 (cont'd): Implementation via Securities

Gap: How to Implement State-Contingent α_t?

Paper assumes direct contracting on $\alpha_t(z_t)$

Securities Design Problem

Find portfolio $\{\theta_t^S, \theta_t^O(K), \theta_t^B\}$ such that compensation replicates optimal $\alpha_t \sigma dZ_t$

Challenge

- Need path-dependent options to capture z_t
- Restricted stock vesting $\approx$ crude approximation
- Dynamic rebalancing required

Critique 2: Renegotiation and Commitment

Full Commitment Assumption

Principal commits to entire future path $\{\alpha_s, \eta_s\}_{s \geq t}$

Renegotiation-Proofness?

After high z_t (front-loaded region):

- Both parties would benefit from reducing α
- But this destroys ex-ante incentives for q

Recommendation

1. Characterize renegotiation-proof contracts
2. Consider limited commitment: T-period contracts
3. Explore reputational equilibria

Critique 2 (cont'd): Renegotiation-Proof Implementation

Renegotiation Constraint

At each t, require:

$$V(Q_t, z_t) + W_t \geq \max_{z'} V(Q_t, z') + W(z'|z_t)$$

Characterizing $\mathcal{Z}^{RP}$

Renegotiation-proof set satisfies:

- Contains z^* (optimal starting point)
- Convex (mixture contracts feasible)
- $v'(z) + W'(z) \leq 0$ for all $z \in \mathcal{Z}^{RP}$

Conjecture

Less back-loading, less front-loading, more compression

Critique 3: CARA-Normal Limitations

Current Setup

- CARA utility
- Normal shocks
- Enables single state z_t

Real World

- CRRA more realistic
- Jump risks matter
- State-dependent volatility

Technical Extension

With CRRA utility $u(c) = \frac{c^{1-\rho}}{1-\rho}$:

$$z_t = \mathbb{E}_t \left[\int_t^\infty e^{-(r+\delta)(s-t)} (r+\delta) \left(\frac{c_t}{c_s} \right)^\rho \alpha_s ds \right]$$

Now depends on entire consumption path!

Critique 3 (cont'd): Breaking CARA-Normal Framework

Suggested Approach: Perturbation Methods

Approximate around CARA solution:

$$V^{\mathrm{CRRA}}(Q, W, z) \approx V_0(Q, z) + \rho V_1(Q, W, z) + \frac{\rho^2}{2} V_2(Q, W, z)$$

Conjecture

Wealth effects $\rightarrow$ state-dependent risk aversion $\rightarrow$ procyclical duration

Recommendation: Numerical analysis with projection methods

Critique 4: Stochastic Volatility and Regime Shifts

Extension to Time-Varying Risk

Let σ_t follow CIR process

New Trade-offs

1. **Volatility timing**: Should PPS vary with σ_t?
2. **Hedging demands**: Agent's precautionary motives
3. **Real option effects**: Wait for low σ to invest in Q?

Conjecture

High volatility states $\rightarrow$ More front-loaded contracts

Critique 5: Multiple Tasks and Spillovers

Corporate Reality (Zechner-Dittmann, 2004)

- Multiple divisions/projects
- Effort spillovers across tasks
- Capital budgeting interactions

Extension

$$dY_t^i = \left(a_t^i + (r+\delta)Q_t^i + \theta \sum_{j \neq i} Q_t^j \right) dt + \sigma_i dZ_t^i$$

New questions:

- How does θ (spillover) affect optimal duration?
- Should contracts be project-specific or firm-wide?

Critique 5 (cont'd): Multiple Agents and Tournaments

N-Agent Extension

Each agent i chooses (a_t^i, q_t^i) affecting:

$$dY_t^i = (a_t^i + (r+\delta)Q_t^i)dt + \sigma dZ_t^i + \sigma_c dZ_t^c$$

Relative Performance Evaluation

$$\alpha_t^i = \phi(z_t^i) + \psi(z_t^i)\left(Y_t^i - \frac{1}{N-1}\sum_{j\neq i} Y_t^j\right)$$

Prediction: RPE weakens incentive for durable investment

Critique 6: Empirical Testing and Identification

Fundamental Challenge

z_t is not directly observable!

Proposed Empirical Strategy

1. **Structural estimation**:
   - Use cash flow dynamics to back out (z_t, Q_t)
   - GMM on moments
2. **Proxy variables**:
   - Vesting schedules $\rightarrow$ proxy for back/front loading
   - Patent citations $\rightarrow$ proxy for Q_t
3. **Natural experiments**:
   - Regulatory shocks to contract space

Additional Critique: Learning and Experimentation

Unknown Productivity

Let true cash flow process be:

$$dY_t = (\theta + a_t + (r+\delta)Q_t)dt + \sigma dZ_t$$

where $\theta \sim \mathcal{N}(\mu_0, \tau_0^{-1})$ unknown

Filtering Problem

Principal's belief: $\hat{\theta}_t \sim \mathcal{N}(\mu_t, \tau_t^{-1})$

$$d\mu_t = \frac{\tau_t}{\sigma}(dY_t - (\mu_t + \hat{a}_t + (r+\delta)\hat{Q}_t)dt)$$

New Incentive to Manipulate

Agent can boost a_t early $\rightarrow$ inflated beliefs

Additional Critique: Continuous vs. Discrete Time

Discrete-Time Analog

$$
\begin{aligned}
Q_{t+1} &= (1-\delta)Q_t + q_t \\
Y_t &= a_t + (r+\delta)Q_t + \sigma\varepsilon_t
\end{aligned}
$$

New Complications

- Agent observes ε_t before choosing a_t
- Timing of information arrival matters
- Richer message spaces possible

Summary: Mathematical Contributions and Gaps

Strong Points

- Elegant state variable reduction via z_t
- Clear characterization of optimal policies
- Novel proof technique for double deviations
- Rigorous verification arguments

Areas for Theoretical Development

1. Numerical methods for non-CARA cases
2. Asymptotic analysis as $\sigma \to 0$ or $\sigma \to \infty$
3. Optimal stopping/switching between contracts
4. Mean field games with many agents

Summary and Future Directions

Paper's Contributions

- Challenges conventional wisdom on myopia
- Elegant mathematical framework
- Rich dynamic implications

Key Areas for Development

1. Relax CARA-normal framework
2. Address implementation/renegotiation
3. Multiple agents and tasks
4. Empirical identification strategy
5. Corporate finance applications (capital structure, M&A)