# Demand Disagreement
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# Motivation I

- Models I : Disagreement about economic fundamentals implies that asset returns highly correlated with shocks to fundamentals
- Models II: Disagreement is highly correlated with disagreement about future asset returns.
- Data I: economic fundamentals not highly correlated with shocks to fundamentals – Correlation Puzzle
- Data II: disagreement about future interest rates is not spanned by the yield curve, macroeconomic fundamentals, and disagreement about macroeconomic fundamentals and this information has substantial predictive power for excess bond returns [Giacoletti, Laursen, and Singleton (2020)] – Disagreement Correlation Puzzle
- Need to find something else for agents in theoretical models to disagree about!
- Basak (2000) opens to the door to a way out: extraneous risk

# Motivation II

- This paper takes a stand on which 'extraneous' risk to model: two types of agent (a & b) who disagree about long-run mean of the fraction of type-a agents.

**Demand Disagreement**

# Heterogeneous Preferences and Beliefs

- Log utility consumers
- Fraction of agents who are Type a: $\alpha_t$ with time discount rate $\rho_a = 0$ and beliefs $\mathbb{P}^a$
- Fraction of agents who are Type b: $1 - \alpha_t$ with time discount rate $\rho_b = 0.05$ and beliefs $\mathbb{P}^b$
- Disagreement about long-run mean of $\alpha_t$: $\alpha_t = 1/(1 + e^{-l_t})$

$$dl_t = \kappa \left( \bar{l}^i - l_t \right) dt + \sigma_l \underbrace{dZ^i_{\alpha,t}}_{\text{demand shock}} , \quad i \in \{a, b\} \tag{1}$$

where $Z^i_{\alpha,t}$ is a Brownian motion under the belief of agent $i$ denoted by $\mathbb{P}^i$.

- $\bar{l}^a > \bar{l}^b$, level of disagreement, $\Delta$

$$Z^a_{\alpha,t} - Z^b_{\alpha,t} = \Delta t, \tag{2}$$

where

$$\Delta = \frac{\kappa}{\sigma_l}(\bar{l}^a - \bar{l}^b) \tag{3}$$

# OLG as in Blanchard (1985) – Stationarity

- Investors enter the market (birth) and then leave randomly (death).
- Time spent trading, $x \sim \exp(\nu)$. Density $\nu e^{-\nu x}$.
- Mean time spent trading is $\nu^{-1} = 50$ years.
- Current time, $t$ and time of entering market is $s$, so time spent trading is $t - s$.
- total mass of agents trading at time $t$ is

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} ds = 1. \tag{4}$$

- Unrealistic, but tractable
- Just a well-chosen device used to generate long-run heterogeneity (see also Garleanu & Panageas (2015)) – how stationarity is acheived unimportant for main message of paper

# Individual Endowments

- Each agent receives an endowment flow $Y$ from birth till death, with dynamics

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dZ_{Y,t} \tag{5}$$

- Endowment flow received by an agent is independent of time alive – all agents which are alive receive the same endowment flow and mass of agents is 1.
- Aggregate endowment is $Y$.

# Disagreement and Trading

- Locally risk-free bond in zero net supply
- Supply asset: an infinitely lived risky asset that is locally perfectly correlated with the supply shock
- Demand asset: an infinitely lived risky asset that is locally perfectly correlated with the demand shock
- a life insurance/annuity contract

Markets are dynamically complete allowing speculation driven by different beliefs – does not matter how this is achieved – just find type-specific SDF's and find value of claim to aggregate endowment

# Wealth

- Type-$i$ investor's wealth if born at time $s$
- Wealth at time $s$

$$\hat{W}^i_{s,s} = E^i_s\left[\int_s^\infty e^{-\nu(u-s)}\frac{\xi^i_u}{\xi^i_s}Y_u du\right] \tag{6}$$

- Wealth at time $t > s$ includes additional term

$$\hat{W}^i_{s,t} = \underbrace{W^i_{s,t}}_{\text{financial wealth}} + \underbrace{E^i_t\left[\int_t^\infty e^{-\nu(u-t)}\frac{\xi^i_u}{\xi^i_t}Y_u du\right]}_{\text{PV of future endowment flows}} \tag{7}$$

# Static Optimization Problem I

- Use Lehoczky, Sethi, and Shreve (1985), Cox & Huang (1989)
- Lagrangean for agent of Type $i$ born at time $s$
  - $s$ is the time of birth and $\tau$ is the time of death.

$$L_s^i = \underbrace{E_s^i \left[ \int_s^\tau e^{-\rho^i (u-s)} \ln c_{s,u}^i dt \right]}_{\text{obj. function}} + \tag{8}$$

$$\overbrace{\underbrace{\kappa_s^i}_{\text{depends on time of birth}}}^{\text{Lagrange multiplier}} \underbrace{E_s^i \left[ \int_s^\tau \frac{\xi_u^i}{\xi_s^i} (c_{s,u}^i - Y_u) du \right]}_{\text{static budget constraint}} \tag{9}$$

# Static Optimization Problem II

- Death arrives according to a Poisson process with intensity $\nu$ – obtain infinite horizon problem with initial time $s$

$$L_s^i = \underbrace{E_s^i\left[\int_s^\infty e^{-(\rho^i+\nu)(u-s)} \ln c_{s,u}^i du\right]}_{\text{obj. function}} + \kappa_s^i E_s^i\left[\int_s^\infty e^{-\nu(u-s)} \frac{\xi_u^i}{\xi_s^i}(c_{s,u}^i - Y_u)du\right] \tag{10}$$

$$\boxed{\frac{c_{s,u}^i}{c_{s,t}^i} = e^{-\rho^i(u-t)}\left(\frac{\xi_u^i}{\xi_t^i}\right)^{-1}} \tag{11}$$

- Type $i$ optimal time-$u$ consumption flow relative to time-$t$ consumption flow is given in terms of intertemporal MRS of Type $i$ investors between times $u$ and $t$ and the time discount rate of Type-$i$ investors

# Static Optimization Problem III

- Optimal consumption in terms of wealth
  Subsitute (11) into

$$E_t^i\left[\int_t^\infty e^{-\nu(u-t)}\frac{\xi_u^i}{\xi_t^i}c_{s,u}^i du\right] = \hat{W}_{s,t}^i \tag{12}$$

$\Rightarrow$

$$c_{s,u} = (\rho^i + \nu)\hat{W}_{s,u}^i \tag{13}$$

- Patient investors consume less out of their wealth than impatient investors

# Consumption Shares

- State variables:
  - Fraction of agents of Type a: $\alpha_t$
  - Consumption share of agents of Type a: $f_t$

$$f_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \alpha_s c_{s,t}^a ds \tag{14}$$

# SDF I

- Use FOC (11) in combination with market clearing to obtain SDFs

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} (\alpha_s c_{s,t}^a + (1-\alpha_s) c_{s,t}^b) ds = Y_t \tag{15}$$

# SDF II

**Theorem 1**

*The dynamics of the investor specific state price densities in equilibrium are*

$$d\xi_t^i = -r_t\xi_t^i dt - \theta_{Y,t}\xi_t^i dZ_{Y,t} - \theta_{\alpha,t}^i\xi_t^i dZ_{\alpha,t}^i, \quad i = a, b$$

*The equilibrium risk-free rate is*

$$r_t = f_t\rho^a + (1 - f_t)\,\rho^b + \nu\left(1 - \alpha_t\beta_t^a - (1 - \alpha_t)\,\beta_t^b\right) + \mu_Y - \sigma_Y^2$$

*where* $\beta_t^i = \left(\nu + \rho^i\right)\phi_t$. *The wealth-consumption ratio is*

$$\phi_t = \frac{f_t}{\rho^a + \nu} + \frac{1 - f_t}{\rho^b + \nu}$$

*The equilibrium market prices of supply and demand shock risks are*

$$\theta_{Y,t} = \sigma_Y, \quad \theta_{\alpha,t}^a = \Delta\,(1 - f_t)\,, \quad \theta_{\alpha,t}^b = -\Delta f_t$$

# SDF III

- demand disagreement is priced
- price of risk for demand disagreement is stochastic – depends on $f$

# Stock Market

- Excess volatility beyond fundamentals and weakly correlated with fundamentals

$$\sqrt{\sigma_Y^2 + \sigma_{\alpha,t}^2} \tag{16}$$

where

$$\sigma_{\alpha,t} = \frac{\phi^a - \phi^b}{\phi_t} f_t(1 - f_t)\Delta \geq 0 \tag{17}$$

- Time-varying risk premium: swings from positive to negative

# Main Idea

- Demand disagreement: disagreement about factors driving sources of demand, such as investor composition
- Just brilliant. Lots of potential.

# Quantifying disagreement dynamics

- How do we gauge realism of parameters for dynamics of $I_t$?
- How volatile is demand?
- Suggestion: connect your equations for asset demands to Koijen & Yogo (2020).

# Match Motivation & Results

- Right now:
  - Motivation from bond markets: disagreement about future interest rates is not spanned by the yield curve, macroeconomic fundamentals, and disagreement about macroeconomic fundamentals and this information has substantial predictive power for excess bond returns [Giacoletti, Laursen, and Singleton (2020)]
  - Results in paper are about equity market
- Suggestions
  - explore whether [Giacoletti, Laursen, and Singleton (2020)] is true for equity markets.
  - explore bond prices more fully in the model in addition to stock market

# How quantitative do you want to be?

- Correlation Puzzle in the Data

| Horizon | Dividends | Consumption |
|---|---|---|
| | Stock Returns | |
| 1 year | 0.08 | 0.00 |
| 5 year | 0.46 | 0.28 |
| 10 year | 0.65 | 0.11 |
| | Trading Volume | |
| 1 year | 0.21 | 0.29 |
| 5 year | 0.35 | 0.38 |
| 10 year | 0.31 | 0.40 |
| | Interest Rate | |
| 1 year | 0.26 | $-0.16$ |
| 5 year | 0.20 | 0.04 |
| 10 year | 0.24 | 0.06 |

- Why not price an exogenous dividend flow (calibrated to the data) to value the stock market?

# Important Results First

- Section IV:E on page 40 is where the theoretical results start getting interesting.
- Paper is not about derivatives – discussion about which zero supply assets are traded not needed
  - Get straight to SDF
  - Show results for asset your are interested in

# Summary

- Basic idea: demand disagreement. Great!
- Need to connect to new work about asset demand
- Which assets are you really interested in? Equities, bonds or both?