Dynamic Equilibrium in an Economy with an Illiquid Stock Market

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Big Picture

- In real world – transaction costs
- How do transaction costs impact asset prices in general equilibrium
  1. Not much impact on level of prices: Vayanos (1999) – exchange economy, proportional transaction costs, CARA utility
  2. Higher risk premia: Amihud and Mendelson (1986)

Even Bigger Picture

- John Maynard Keynes, The General Theory of Employment, Interest and Money, 1936.

  The Jobber's 'turn', the high brokerage charges and the heavy transfer tax payable to the Exchequer, which attend dealings on the London Stock Exchange, sufficiently diminish the liquidity of the market (...) to rule out a large proportion of the transaction characteristic of Wall Street. The introduction of a substantial government transfer tax on all transactions might prove the most serviceable reform available, with a view to mitigating the predominance of speculation over enterprises in the United States.

- Milton Friedman, Essays in Positive Economics, 1953.

  Despite the prevailing opinion to the contrary, I am very dubious that in fact speculation in foreign exchange would be destabilizing. Evidence from some earlier experiences and from current free markets in currency in Switzerland, Tangiers, and elsewhere seem to me to suggest that, in general, speculation is stabilizing rather than the reverse, though the evidence has not yet been analyzed in sufficient detail to establish this conclusion with any confidence.

Even Bigger Picture

- Tobin tax
    - securities transactions tax imposed specifically on foreign exchange transactions and possibly their derivatives
- Recent Credit Crunch: should financial transactions be taxed more heavily?
    - 2009 – H.M. Treasury looked at implications of adopting an FTT for financial markets
    - 2010 – European Parliament released a study of financial transaction taxes and charged the European Commission with developing plans for a European financial transactions tax

This paper

- Looks at equilibrium asset prices in an exchange economy with one risky stock and a risk-free asset
  - one class of agent faces *linear-convex* transaction costs (investors), and
  - another class of agents (market makers) faces search costs and receives transactions costs paid by investors
- interesting results on
  - equity risk premium – can be negative
  - stock market volatility higher in presence of transaction costs
  - transaction costs lead to more extreme movements in stock prices

Model details

- Dividend $dD_t = \mu_D dt + \sigma_D dW_t$
- Stock is claim to dividend $dS_t = \mu_{S,t} dt + \sigma_{S,t} dW_t$
- agents max expected utility of terminal wealth of at date T – can set risk-free rate to zero
- Market makers, $m \in \{1, \ldots, M\}$: $\max_{N^m_{1,t}} E_0[u^m_1(X^m_{1,T})]$

$$\begin{aligned} dX^m_{1,t} &= N^m_{1,t}\mu_{S,t}dt + N^m_{1,t}\sigma_{S,t}dW_t + \underbrace{\left(tc^m_t - sc^m_t\right)}_{=0} dt \\ X^m_{1,t} &= N^m_{1,t}S_t + B^m_{1,t} \end{aligned}$$

- Investors, $i \in \{1, \ldots, I\}$: $\max_{N^i_{2,t}} E_0[u^i_2(X^i_{2,T})]$

$$\begin{aligned} dX^i_{2,t} &= (N^i_{2,t}\mu_{S,t} - (\underbrace{\alpha_1|u^i_{2,t}|}_{\text{prop. cost}} + \underbrace{\alpha_2|u^i_{2,t}|^{1+\epsilon}}_{\text{convex cost}}))dt + N^i_{2,t}\sigma_{S,t}dW_t \\ dN^i_{2,t} &= u^i_{2,t}dt \\ X^i_{2,t} &= N^i_{2,t}S_t + B^i_{2,t} \end{aligned}$$

Comments on Model

- market makers essentially face no transaction costs $sc = tc$
- market makers and investors have different preferences: purely for technical convenience

$$\begin{aligned} u_m(x) &= \ln x \\ u_i(x) &= -e^{-\frac{1}{\gamma}x} \end{aligned}$$

  - economics would be clearer with identical preferences
- proportional costs – no trading zone, alone does not impact level of asset prices much
- convex costs –trade is infrequent, no of shares held by investor is locally risk-free $dN^i_{2,t} = u^i_{2,t}dt$
  - rare in transaction costs lit., more common in RBC models
  - would it be ok to just have convex costs?

Important intuition

- Market maker

$$N^m_{1,t} = X^m_{1,t} \frac{\mu_{S,t}}{\sigma^2_{S,t}}$$

  - wealth effect – as S increases, buy more stock (dominant effect)
  - substitution effect – as S increase buy less stock
- Investor – no wealth effect, just substitution effect, but small because of transaction costs
  - driven by log preferences?
- Market makers impact prices, investors impact direction of trade

Negative risk premium I

- If market maker is short stock, and stock prices goes up, market maker want to buy more stock (wealth effect), and investor wants to sell (subn effect)
- stock price rises further, market maker worse off because of being short
- stock price change positively correlated with marginal utility of market maker
- stock price change positively correlated with change in SPD (is this true? check!)

$$\mu_{S,t} = -E_t \left[dS_t \frac{d\xi_t}{\xi_t} \right]$$

- negative risk premium

Negative risk premium II

- Arises when aggregate market maker is short, i.e.

$$N_{1,t} = \frac{1}{M} \sum_{m=1}^{M} N_{1,t}^{m} < 0$$

- Does this occur often, or is it rare?
- Is there a lower bound on $N_{1,t}$?

Stock market volatility

- Relative to model with no transaction costs, stock market vol. is higher
  - $N_{1,t} > 0$ and large, $\mu_{S,t} > 0$
  - Increase in $D \longrightarrow$ increase in S
  - market maker buys more stock
  - S increases further (amplification of initial shock)
- Only high when $N_{1,t}$ is large in magnitude
- How do we know there are no endogenous limits to $|N_{1,t}|$.

Extreme movements on stock price

- Suppose investors are long and market makers are short
  - stock volatility higher than is possible in liquid economy
  - as a function of N_1 volatility has a U shape, like a smile
  - probability of extreme price movements higher than in liquid economy
- Comments
  - simulate paths of stock price
  - estimate probability of large downward shifts in stock price and compare with model with no transaction costs (liquid economy)
  - think about applications to option pricing (implied volatility smile)

Boundedness of market price of risk

- SPD, local exponential martingale

$$\begin{aligned} \xi_t &= e^{-\frac{1}{2}\int_0^t \left(\frac{\mu_{S,u}}{\sigma_{S,u}}\right)^2 du - \int_0^t \frac{\mu_{S,u}}{\sigma_{S,u}} dW_u} \\ \frac{d\xi_t}{\xi_t} &= \underbrace{\frac{\mu_{S,t}}{\sigma_{S,t}}}_{\text{market price of risk}} dW_t \end{aligned}$$

- market price of risk bounded $\longrightarrow$ local martingale is a true martingale, existence of unique equivalent martingale measure, and rules out arbitrage opportunities
- is market price of risk (Sharpe ratio) unbounded?
- if so, may or may not be arbitrage opportunities. Need to look at Delbaen and Schachermeyer (1998)

Concluding comments

- Interesting results
    - Negative risk premium
    - More illiquidity $\rightarrow$ more extreme price movements
- Needs further work
    - Small changes to model
        - Drop preference heterogeneity?
        - Drop proportional cost?
    - Compute correlation of changes to SPD with changes in stock price
    - Simulate the model
        - estimate how often negative risk premium occurs
        - estimate how often large downward stock price movements (crashes) occurs
    - Is the Sharpe Ratio bounded?
    - Is there a hidden bound on $|N_1|$?
    - Further work: implications for option pricing (vol smile)