Hansen-Jagannathan Bounds with Convenience Yields by Zhengyang Jiang and Robert J. Richmond

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Sharpe Ratios on US Treasuries and the SDF I

Table: **Treasury Sharpe Ratios across Maturities** The Avg Excess Return is $\mathbb{E}[R] - R^f$ and the Sharpe Ratio is $\left(\mathbb{E}[R] - R^f\right)/\sigma(R)$. All values are annualized. Data are from CRSP and Bloomberg.

	< 6m	$6-12$m	$1-2$y	$2-3$y	$3-4$y	$4-5$y	$5-10$ y
$\mathbb{E}[R] - R^f$ (%)	0.30	0.58	0.92	1.32	1.64	1.74	2.19
$\sigma(R)$(%)	0.39	1.25	2.33	3.64	4.57	5.44	6.78
$\frac{\mathbb{E}[R]-R^f}{\sigma(R)}$	0.77	0.47	0.40	0.36	0.36	0.32	0.32

- Table 1 has interesting implications for the martingale component in the SDF
  - Sharpe ratio higher than for SP500!
  - Martingale needs to be very volatile – even more so than implied by SP500 data

$$M_t = R^f_{0,t}\mathcal{M}_t \tag{1}$$

Sharpe Ratios on US Treasuries and the SDF II

- Existing term structure literature, Duffee (2011)
  - conditional Sharpe ratio implied by 4 or 5-factor affine term structure model is really high (astronomical)
  - result of overfitting due to the flexibility permitted by the four or five factors
  - proposed solution: impose a good deal bound on the maximum Sharpe ratio in model estimation

This paper: Convenience Yields

- Safe assets have convenience yields
  - convenience yield: reduction in bond yields relative to the benchmark rates that investors are willing to give up to hold safe asset (Krishnamurthy and Vissing-Jorgensen (2012); Nagel (2016))
  - bond return is risk-free return plus a convenience yield

$$R^f e^{\lambda} \tag{2}$$

$$lnR^f + \lambda \tag{3}$$

  - no longer need such a super volatile martingale component in SDF

Convenience Yields and HJ Bounds I

- Price risky asset

$$
\begin{aligned}
E[MR] = 1 \Rightarrow E[M]E[R] + Cov[M, R] &= 1 && (4)\\
\Rightarrow E[R] &= (E[M])^{-1} - Cov[M(E[M])^{-1}, R] && (5)\\
\Rightarrow E[R] - (E[M])^{-1} &= -Cov[M(E[M])^{-1}, R] && (6)
\end{aligned}
$$

- Note that $M(E[M])^{-1}$ is a martingale, $\mathcal{M} = M(E[M])^{-1}$

$$
\begin{aligned}
E[R] - (E[M])^{-1} &= -Cov[\mathcal{M}, R] && (7)\\
E[R] - (E[M])^{-1} &= -Corr[\mathcal{M}, R]\sigma[\mathcal{M}]\sigma[R] && (8)
\end{aligned}
$$

Convenience Yields and HJ Bounds II

- Use fact that $|Corr[\mathcal{M}, R]| \leq 1$

$$\boxed{\left|\frac{E[R] - (E[M])^{-1}}{\sigma[R]}\right| \leq \sigma[\mathcal{M}]} \tag{9}$$

- But what is $E[M]$?

Convenience Yields and HJ Bounds III

- Traditional approach

$$E[MR^f] = 1 \Rightarrow E[M] = \frac{1}{R^f} \tag{10}$$

$$\boxed{\left|\frac{E[R] - R^f}{\sigma[R]}\right| \leq \sigma[\mathcal{M}]} \tag{11}$$

- Standard form of HJ bound

Convenience Yields and HJ Bounds IV

- This paper: price safe asset with a convenience yield

$$E[MR^f e^{\lambda}] = 1 \Rightarrow E[M] = \frac{1}{R^f} e^{-\lambda} \tag{12}$$

$$\boxed{\left| \frac{E[R] - R^f e^{\lambda}}{\sigma[R]} \right| \leq \sigma[\mathcal{M}]} \tag{13}$$

- Obtain HJ bound with convenience yield

Measuring the convenience yield

- difference between Repo rates and treasury yield (Nagel, 2016)
- Repo rate approximated by scaled CD rate

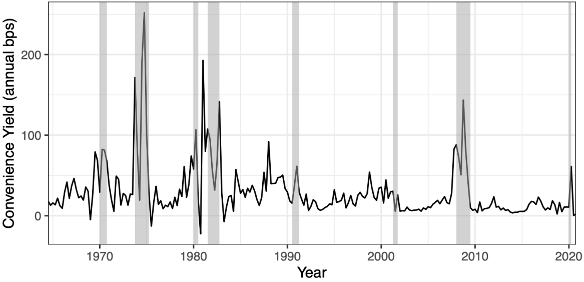

Figure 3: Convenience Yield on Short Term Treasuries over Time

Annual convenience yields (bps) on 3-month U.S. Treasuries. Convenience yields are measured as the difference between 3-month scaled CD rates and yields on 3-month U.S. Treasuries. CD rates are scaled by the ratio of 3-month GC repo rates and 3-month CD rates over the sample where both are available. Data are from CRSP and Bloomberg.

Term Structure Model I

- nominal SDF, $M^{\$}_{t+1} = \exp\left(m^{\$}_{t+1}\right)$,

$$m^{\$}_{t+1} = -y^{\$}_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1},$$

Λ_t is an $N \times 1$ vector

$$\Lambda_t = \Lambda_0 + \Lambda_1 z_t.$$

$N \times 1$ vector Λ_0: average prices of risk while the $N \times N$ matrix Λ_1: time varying components of risk premia

Term Structure Model II

$$z_t = \Psi z_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t$$

$$z_t = \left[\pi_t, x_t, y_t^{\$}(1), yspr_t \right]'$$

- log inflation π_t
- log GDP growth x_t
- log 3-month nominal yield $y_t^{\$}(1)$
- $yspr_t$ is difference between log 5-year nominal yield and log 3-month nominal yield

Term Structure Model III

- impose a good deal bound

$$\theta_t = \sqrt{\text{var}_t\left(m^{\$}_{t+1}\right)} = \sqrt{\Lambda'_t \Lambda_t}.$$

$$\text{mean}\left(\theta_t\right) < b,$$

- Convenience yield in Treasuries improves fit to bond prices
- Constant convenience yield of 60 b.p. gives best fit
- Stochastic convenience yield provides better fit than constant convenience yield

Summary

- Treasury convenience yield impacts HJ bound logic
- TSM + data $\rightarrow$ unconditional convenience yield is between 10 and 100 bps (can say this with some confidence)
- TSM + data $\rightarrow$ convenience yield is stochastic

I: Why do you need a good deal bound?

- Duffee (2001) finds that sample mean of conditional maximum Sharpe ratios produced by an estimated five-factor model is on the order of 10^{30} (astronomical!)
- Therefore he imposes a constraint on maximum Sharpe ratios
- HJ logic implies convenience yield should already make the volatility of the SDF reasonable
- If the convenience yield is relevant empirically, you should not need to constrain Sharpe ratios in your estimation
- Internal logic of paper needs to be consistent

II: Other work

- Compare your results on convenience yield estimates with Binsbergen, Diamond, Grotteria (2022)
- Your mean estimates look similar. Provide further comparison

III: Other Asset Classes

- Can you reestimate your model using stock market data **and** bond data?
- Treasury convenience yields have some connection with SP500, especially in bad times
- How would the risk prices change if you added SP500 return data?
- Can you get an SDF that prices bonds and the SP500 without constraining Sharpe ratios?

IV: Which other assets have a convenience yield?

- Cochrane (2002) argues that during the dotcom bubble, there was an intense demand for short-term trading in tech stocks. Few such shares were available for trading – the available shares had a convenience yield.

V: Rethinking asset pricing: time-varying convenience yields vs time-varying risk prices

$$\frac{dM_t}{M_t} = -r_t dt - \lambda_t dt - \Theta_t dZ_t \tag{14}$$

- How important is time variation in λ_t versus time variation in Θ_t?

VI: Portfolio Choice Implications

- Sangvinatsos & Wachter (2005): desire to hedge changes in term premia generates large hedging demands for long-term bonds.
- How important is the desire to hedge changes in convenience yields for portfolio choice?

Will he be happy?

Conclusion

$$\boxed{\left| \frac{E[R] - R^f e^{\lambda}}{\sigma[R]} \right| \leq \sigma[\mathcal{M}]} \tag{15}$$

- Convenience yields rear their head again
- Why constrain Sharpe ratios in the estimations?
- Can you price both bonds and stocks?
- What are the implications of convenience yields for bond portfolio choices?
- Are there convenience yields in other asset classes?